Problem 1

Let S be the union of the set of all points inside a regular nonagon with side length 2 units and the set of all points less than 1 unit away from a point on the perimeter of the nonagon. What, in units, is the perimeter of S?

Problem 2

Let a, b, c, and d be positive real numbers such that $a + b + c + d = 10$. Find the maximum value of $ab^2c^3d^4$.

Problem 3

Find all the integer roots of

$$x^4 + 5x^3 + 9x^2 - x - 14 = 0.$$

Enter all the integer roots, separated by commas.

Problem 4

The symbols $\triangle$, $\square$, $\diamond$, $\clubsuit$ represent four different integers from 1 to 9. Using the equations below, what is the value of $\square$?

$$\begin{aligned}
\triangle + \square &= \clubsuit \\
\triangle + \triangle &= \diamond + \diamond + \diamond + \diamond + \diamond \\
\triangle + \triangle &= \clubsuit + \diamond.
\end{aligned}$$

Problem 5

What is $\left(4\frac{5}{8}\right)^{55} \cdot \left(\frac{8}{37}\right)^{55}$?

Problem 6

A pirate searches seven islands for buried treasure. If each island has a $\frac{1}{5}$ chance of having treasure, what is the probability that exactly 4 of the islands have treasure?

Problem 7

A regular hexagon can be divided into six equilateral triangles. If the perimeter of one of the triangles is 21 inches, what is the perimeter, in inches, of the regular hexagon?

Problem 8

You have seven bags of gold coins. Each bag has the same number of gold coins. One day, you find a bag of 53 coins. You decide to redistribute the number of coins you have so that all eight bags you hold have the same number of coins. You successfully manage to redistribute all the coins, and you also note that you have more than 200 coins. What is the smallest number of coins you could have had before finding the bag of 53 coins?

Problem 9

Let

$$f(n) = \begin{cases} \lfloor n \rfloor & \text{if } n \geq 4, \\ \lceil n \rceil & \text{if } n < 4. \end{cases}$$

Find $f\left(\frac{\pi}{3}\right) + f(\sqrt{45}) + f(8^{2/3})$.

Problem 10

What is the smallest positive integer n such that all the roots of $z^4 + z^2 + 1 = 0$ are n^{th} roots of unity?

Source: HuggingFaceH4/MATH-500. Random sample balanced by subject and difficulty.